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VARIABILITY OF THE CHOICE OF THE MATHEMATICAL MODELS IN APPLIED SECURITY PROBLEMS

The variability of the choice of mathematical models for solving applied safety problems allows to substantiate the methods of nonlinear mathematical programming to increase the accuracy of the implementation of the main optimization task of increasing the quality of the biotechnological process of laser embryo division. This is achieved thanks to the implementation of several applied optimization mathematical models, which are partial cases of the main optimization problem. The article presents 9 applied optimization mathematical models of the process of laser action on the embryo. According to the authors of this article, the most meaningful is the applied optimization model for minimizing the number of thermally injured cells. Other applied optimization mathematical models presented in this article are auxiliary and contain general recommendations for optimizing the technical parameters of laser emitters and improving the quality of laser embryo division.

Optimization of technical parameters is possible only after calculating the values of the temperature of laser action, that is, after solving a nonlocal boundary value problem with a system of heterogeneous, nonlinear, multidimensional partial differential equations with conjugation conditions in the layers of the embryo. To substantiate the correctness of the boundary value problem, the authors applied specialized methods of nonlinear mathematical programming based on the theory of pseudo-differential operators over the space of generalized functions of slow power growth. Taking into account the specific features of the boundary value problem and applied optimization mathematical models, the calculation of the values of the temperature field in the embryo was carried out by the methods of separated variables and uncertain coefficients. Applying the calculated temperatures of laser heating in the layers of the embryo, a mathematical model of minimizing the number of thermally injured embryos was implemented.

Key words: *variability, safety environment, bioobject, optimization mathematical models, mathematical programming problems.*

Formulation of the problem. Currently, the greatest interest is in mathematical models, methods for implementing the parameter optimization process and their application to solving particular problems. Mathematical models for calculating the temperature of laser exposure on a multilayer microbiological object (embryo) are non-local boundary value problems for a system of partial differential equations. The correctness of the calculated mathematical models (boundary value problems) entails the correctness of the main problem of optimizing the parameters of laser division of the embryo.

In the article, based on the variable analysis of mathematical methods and models, the problem of designing a safe environment for the embryo under the influence of laser radiation sources is solved. For this, the authors provide 9 applied optimization mathematical models and a calculated mathematical model of the process of laser action on the embryo. According to the authors of this article, the implementation of mathematical models for minimizing the number of thermally injured embryos and avoiding the temperature field from its predetermined value will allow to increase the accuracy and speed of optimization of the selected technical parameters of laser emitters, which will increase the viability of cells during laser division of embryos.

Analysis of recent research and publications. Having analyzed the phase composition of cast iron alloys and using specialized optical-mathematical methods, the article [1] developed a structural approach for controlling the production of mechanisms and their use in heavy engineering. Proposed mathematical methods and designed devices for increasing the efficiency of ensuring biotechnological processes in animal husbandry [2]. The work [3] analyzed the advantages and disadvantages of the most well-known currently existing technologies for laser division of embryos, and solved certain applied problems of improving the quality of laser division of embryos for embryo transplantation. Having analyzed some existing cyber threats to electrical systems in Ukraine, article [4] proposes methods for increasing the efficiency of electrical systems, which are based on the theory of systems with distributed parameters. The authors of publications [5–7] conducted fundamental research on computational and applied optimization mathematical models in applications to systems with distributed parameters, in particular to solving optimization problems for thermophysical, electrical and mechanical systems.

Using dynamic models and methods of economic modeling in articles [8, 9], strategies for optimal management of technical innovations to minimize the effects of risks in the production activity of domestic agricultural enterprises were developed. The authors of articles [10, 11] studied mechanisms for attracting additional investments from domestic and foreign investors to reduce the influence of risk factors in the production activities of domestic agricultural enterprises.

Task statement. Determine and justify the conditions for creating a safe environment for the biological object when using innovative research methods.

Outline of the main material of the study. Let's consider some applied optimization mathematical models.

Mathematical model 1. It is necessary to minimize the difference between the values of the temperature field at given points of the multilayer system (Fig. 1) and pre-specified permissible values of the temperature field:

$$\min_{z^*\in Z} \max_{\substack{(x_i,y_i,z_i)\in\Omega_i \\ i=1,N \\ t\in[t_0;t^*]}} \left| T_i\left(x_i,y_i,z_i,t,z^*\right) - T^* \right|, \quad (1)$$

where $T_i\left(x_i,y_i,z_i,t,z^*\right)$ – temperature field;
T^* – permissible temperature field values;
$\left(x_i,y_i,z_i\right)\in\Omega_i\in\Omega^*, i=1,..,N$ – area of multilayer microbiological material;
z^* – vector of parameters of thermal effects on multilayer microbiological material;
t_0 – initial time t;
t^* – final point in time t.

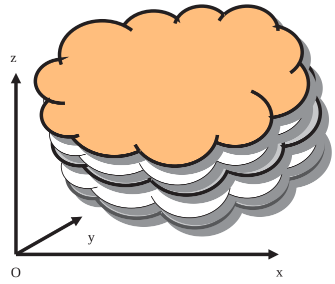

Fig. 1. Multilayer structure of microbiological material

Mathematical model 2. It is necessary to minimize the maximum value, among control points, of the temperature field, find

$$\min_{z^*\in Z} \max_{\substack{(x_i,y_i,z_i)\in\Omega_i \\ i=1,N \\ t\in[t_0;t^*]}} T_i\left(x_i,y_i,z_i,t,z^*\right). \quad (2)$$

Notice, that $T_i\left(x_i,y_i,z_i,t,z^*\right)$ – temperature field of the point area $\left(x_i,y_i,z_i\right)\in\Omega_i\in\Omega^*,\ i=1,..,N$ multilayer microbiomaterial Ω, a z^* – vector of parameters of thermal impact on multilayer microbiomaterial.

Mathematical model 3. It is necessary to minimize, in terms of thermal impact parameters, the maximum value of the temperature field in the area of multilayer microbiological material:

$$\min_{z^*\in Z} \max_{\substack{(x,y,z)\in\Omega^* \\ t\in[t_0;t^*]}} T\left(x,y,z,t,z^*\right). \quad (3)$$

In the above mathematical model $T\left(x,y,z,t,z^*\right)$ is the temperature field of the area of points $\left(x,y,z\right)\in\Omega^*$ multilayer microbiomaterial Ω, t_0 and t^* – start and end times t.

Mathematical model 4. It is necessary to minimize, according to the thermal impact parameters, the maximum value of the temperature field gradient modulus in the area of points Ω^*, multilayer microbiological material Ω, that is, find

$$\min_{z^*\in Z} \left[\max_{\substack{(x,y,z)\in\Omega^* \\ t\in[t_0;t^*]}} \left| grad\ T\left(x,y,z,t,z^*\right) \right| \right]. \quad (4)$$

Wherein $T\left(x,y,z,t,z^*\right)$ is the temperature field of the area of points $\left(x,y,z\right)\in\Omega^*$ multilayer (N-layer) microbiomaterial Ω, a z^* – vector of thermal effect parameters.

Mathematical model 5. In practice, there are often problems when it is necessary to estimate the number of viable and thermally injured embryos. This problem arises during laser division of early elite cattle embryos for further transplantation of embryo parts. In this case, it is necessary to maximize the number of viable embryos, that is, find

$$\sum_{i=1}^{N} T_i\left(x_i,y_i,z_i,t,z^*\right) \to \max_{\substack{(x_i,y_i,z_i)\in\Omega^* \\ t\in[t_0;t^*] \\ z^*\in Z \\ i=1,N}}, \quad (5)$$

where $T_i = \begin{cases} 0, & T_i > T^*, \\ 1, & T_i \le T^*; \end{cases}$

T_i – the value of the temperature field in the area of points of a multilayer (three-layer) microbiological object (embryo) Ω.

Notice, that T^* – this is the permissible temperature at the points belonging to the embryos, t_0 and t^* – start and end times *t*, a z^* – vector of thermal effect parameters.

Mathematical model 6. Consider the problem of controlling the nature of the interlayer distribution of the temperature field:

$$\min_{z^* \in Z} \left| \max_{\substack{(x,y,z)\in N_1 \\ t\in[t_0;t^*]}} T\left(x,y,z,t,z^*\right) - \max_{\substack{(x,y,z)\in N_2 \\ t\in[t_0;t^*]}} T\left(x,y,z,t,z^*\right) \right|, \quad (6)$$

where N_1, N_2 – areas occupied by controlled layers.

In its turn, $T\left(x,y,z,t,z^*\right)$ – temperature field, t_0 and t^* – start and end times *t*, a z^* – vector of laser beam action parameters.

Mathematical model 7. Consider the problem of monitoring thermal stresses arising between layers:

$$\min_{z^* \in Z} \left(\max_{\substack{(x,y,z)\in N_1 \\ t\in[t_0;t^*]}} T\left(x,y,z,t,z^*\right) - \min_{\substack{(x,y,z)\in N_2 \\ t\in[t_0;t^*]}} T\left(x,y,z,t,z^*\right) \right). \quad (7)$$

Wherein $T\left(x,y,z,t,z^*\right)$ is the temperature field of the regions N_1, N_2 adjacent layers of microbiological material Ω.

Mathematical model 8. Consider the problem of minimizing the deviation (deviation) of the temperature field of a microbiological material, when exposed to a laser beam, from a predetermined (desired) distribution of the temperature field in the area of points of the microbiological material:

$$\min_{z^* \in Z} \max_{\substack{(x,y,z)\in\Omega^* \\ t\in\left[t_0;t^*\right]}} \left(\int_{\Omega^*} \left(T\left(x,y,z,t,z^*\right) - T_0\left(x,y,z,t\right) \right)^2 d\Omega^* \right)^{1/2}, \quad (8)$$

where $T_0(x,y,z,t)$ – predetermined (desired) distribution of the temperature field in the region Ω^* microbiological material Ω.

Mathematical model 9. Consider the problem of minimizing the deviation (deviation) of the temperature field of a microbiological material, when exposed to a laser beam, from a predetermined (desired) distribution $T_0(x,y,z,t)$ temperature field on a smooth curve *L* in microbiological material:

$$\min_{z^* \in Z} \max_{\substack{(x,y,z)\in L \\ t\in\left[t_0;t^*\right]}} \left(\int_{L} \left(T\left(x,y,z,t,z^*\right) - T_0\left(x,y,z,t\right) \right)^2 dL \right)^{1/2}. \quad (9)$$

In turn, in the formula (9) $T\left(x,y,z,t,z^*\right)$ – temperature field of points $\left(x_i,y_i,z_i\right)\in L$ smooth curve of multilayer microbiological material Ω.

To implement applied optimization mathematical models, the values of the objective function (temperature field) are required, which are obtained by solving the boundary value problem of the process of laser exposure to the embryo. The main difficulty in parameterizing temperature fields is obtaining an analytical (approximately analytical) or algorithmic (numerical) representation of the solution to the boundary value problem depending on the changeable (during the system synthesis) parameters included in the formulation of the original boundary value problems. In the case where there is an analytical solution to a boundary value problem, which organically includes the required parameters, it is possible to eliminate the time-consuming process of solving a series of similar boundary value problems. Such analytical solutions, unfortunately, are available only for classical domains (segment, circle, circle, ball, sphere). Note that the consideration of any new boundary value problem describing the temperature field in the system under consideration requires overcoming the difficulties associated with the field parameterization stage [12–14].

Calculation mathematical model of the process of laser exposure to an embryo:

$$\begin{cases} \rho_1 c_1 \dfrac{\partial T_1}{\partial t} - \lambda_1 \left(\dfrac{\partial^2 T_1}{\partial r^2} + \dfrac{2}{r_1}\dfrac{\partial T_1}{\partial r} \right) + q_1 = 0, \quad r\in[0;r_1],\ t\in[0;t_1]; \\ \rho_2 c_2 \dfrac{\partial T_2}{\partial t} - \lambda_2 \left(\dfrac{\partial^2 T_2}{\partial r^2} + \dfrac{2}{r_2}\dfrac{\partial T_2}{\partial r} \right) + q_2 = 0, \quad r\in[r_1;r_2],\ t\in[t_1;t_2]; \\ \dots \ \dots \ \dots \ \dots \ \dots \ \dots \\ \rho_N c_N \dfrac{\partial T_N}{\partial t} - \lambda_N \left(\dfrac{\partial^2 T_N}{\partial r^2} + \dfrac{2}{r_N}\dfrac{\partial T_N}{\partial r} \right) + q_N = 0, \quad r\in[r_{N-1};r_N],\ t\in[t_{N-1};t_N], \end{cases} \quad (10)$$

where ρ_e – density coefficient e layer of multilayer (N – layer) microbiomaterial, where $e = 1,..,N$;

c_e – heat capacity coefficient of the e layer of multilayer (N – layer) microbiomaterial;

λ_e – thermal conductivity coefficient of the e layer of multilayer microbiomaterial;

q_e – specific power density of thermal loads in a multilayer microbiomaterial;

$T_e = T_e\left(r,t\right)$ – temperature field of points of the e layer of multilayer microbiomaterial;

r_e – spatial coordinate;

t_e – time parameter.

Dirichlet boundary conditions:

$$\begin{cases} T\left(r,t\right)\big|_{\substack{r=r_0 \\ t=t_0}} = T_0; \\ T\left(r,t\right)\big|_{\substack{r=r_n \\ t=t_n}} = T_n. \end{cases} \quad (11)$$

Equalities in the division of media:

$$\begin{cases} T_1\left(r_1,t_1\right) = T_2\left(r_2,t_2\right), \quad -\lambda_1 \dfrac{\partial T_1}{\partial r} = -\lambda_2 \dfrac{\partial T_2}{\partial r}; \\ T_2\left(r_2,t_2\right) = T_3\left(r_3,t_3\right), \quad -\lambda_2 \dfrac{\partial T_2}{\partial r} = -\lambda_3 \dfrac{\partial T_3}{\partial r}; \\ \dots \ \dots \ \dots \ \dots \ \dots \ \dots \\ T_{N-1}\left(r_{N-1},t_{N-1}\right) = T_N\left(r_N,t_N\right), \quad -\lambda_{N-1} \dfrac{\partial T_{N-1}}{\partial r} = -\lambda_N \dfrac{\partial T_N}{\partial r}. \end{cases} \quad (12)$$

The notations are the same as in the system of differential heat conduction equations (10).

Boundary conditions of heat exchange:

$$\left.\left(\lambda_1 \frac{\partial T_1}{\partial r} - A\left(T_1 - T_{ext}\right)\right)\right|_{r=0} = 0\,, \qquad (13)$$

where λ_1 – thermal conductivity coefficient of the outer layer of a multilayer (N -layer) microbiological material;
A – heat transfer parameter of the outer (outer) layer of a multilayer microbiological material;
T_{ext} – ambient temperature.

Taking into account the specifics of laser action, boundary conditions (13) will look as follows:

$$-\lambda_1 \frac{\partial T_1}{\partial r}(0,t) = qS, \quad 0 \le t \le h\,, \qquad (14)$$

where q – specific heat flux;
S – diameter of heat source.

Let us divide the solution of the differential heat equation from system (10) into the sum of two solutions:

$$T(r,t) = T_{g.h.}(r,t) + T_{p.i.}(r,t)\,, \qquad (15)$$

where $T_{g.h.}(r,t)$ – general homogeneous solution;
$T_{p.i.}(r,t)$ – private inhomogeneous solution.

General homogeneous solution:

$$T(r,t) = u(r)v(t)\,. \qquad (16)$$

Substituting $T(r,t)$ into the equation of system (10), we got:

$$v'(t)u(r) - a(v(t)u''(r) + \frac{2}{r}v(t)u'(r)) = 0\,. \qquad (17)$$

Transformed the differential equation (17):

$$v'(t) = cv(t)\,,\; u''(r) + \frac{2}{r}u'(r) = \frac{c}{a}u(r)\,. \qquad (18)$$

Solving an equation with separating variables and a Fuchs class equation, respectively:

$$v(t) = e^{ct}\,,\; u_1(r) = \sum_{k=0}^{\infty} \frac{c^k r^{2k-1}}{a^k \left((2k)!!\right)^2} \text{ and}$$

$$u_2(r) = \sum_{k=0}^{\infty} \frac{c^k r^{2k}}{a^k \left((2k+1)!!\right)^2}\,. \qquad (19)$$

Let us write down the general homogeneous solution of the differential equation:

$$T(r,t) = \left(c_1 u_1(r) + c_2 u_2(r)\right) e^{ct}\,. \qquad (20)$$

Particular inhomogeneous solution of the heat equation from system (10):

$$T_{ч.н.}(r,t) = -\frac{q_e}{6a} r^2\,. \qquad (21)$$

Using the results obtained, we write the solution to the differential heat equation from the system (10):

$$T(r,t) = T(0,0)e^{ct} \sum_{k=0}^{\infty} \frac{c^k r^{2k}}{a^k \left((2k+1)!!\right)^2} - \frac{q_e}{6a} r^2 g(t)\,. \qquad (22)$$

Having obtained the solution function to the boundary value problem (10)–(14), we proceed to the implementation of the applied optimized mathematical model 5, that is, to minimizing the number of viable cells.

Conclusions. Having studied the variability of applied optimization mathematical models for solving the problem of designing a safe environment for a biological object, the article proposes and substantiates the conditions for improving the quality of the biotechnological process of laser embryo division. Increasing the viability of cells is achieved due to the implementation of the mathematical models proposed in the article. It should be noted that for the optimization of the selected parameters of the biotechnological process, the values of the temperature of the laser action are required, which are obtained after the implementation of non-local boundary value problems of systems of multidimensional equations of thermal conductivity. Taking into account the specific features of applied optimization mathematical models, analytical methods of nonlinear mathematical programming were chosen to calculate the values of the temperature field. Increasing the accuracy of ensuring the control using the technical resources of laser emitters, which is achieved thanks to the use of the layer-by-layer distribution of temperature fields in the embryo obtained in this article, will allow to increase the level of viability of the bioobject during laser division.

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Левкін Д.А., Жерновникова О.А., Синявіна Ю.В., Левкін А.В. ВАРІАТИВНІСТЬ ВИБОРУ МАТЕМАТИЧНИХ МОДЕЛЕЙ У ПРИКЛАДНИХ БЕЗПЕКОВИХ ЗАДАЧАХ

Варіативність вибору математичних моделей для розв'язання прикладних безпекових задач дозволяє обґрунтувати методи нелінійного математичного програмування для підвищення точності реалізації основної оптимізаційної задачі збільшення якості біотехнологічного процесу лазерного ділення ембріона. Це досягається завдяки реалізації декількох прикладних оптимізаційних математичних моделей, які є частковими випадками основної оптимізаційної задачі. В статті наведені 9 прикладних оптимізаційних математичних моделей процесу лазерної дії на ембріон. На думку авторів цієї статті, найбільш змістовною є прикладна оптимізаційна модель мінімізації числа термічно-травмованих клітин. Інші ж наведені в цій статті прикладні оптимізаційні математичні моделі є допоміжними та містять загальні рекомендації щодо оптимізації технічних параметрів лазерних випромінювачів і підвищення якості лазерного ділення ембріона.

Оптимізація технічних параметрів можлива лише після розрахунку значень температури лазерної дії, тобто після розв'язання нелокальної крайової задачі з системою неоднорідних, нелінійних, багатовимірних диференціальних рівнянь в частинних похідних з умовами спряження в шарах ембріона. Для обґрунтування коректності крайової задачі автори застосували спеціалізовані методи з нелінійного математичного програмування, що базуються на теорії псевдодиференціальних операторів над простором узагальнених функцій повільного степеневого зростання. Врахувавши специфічні особливості крайової задачі і прикладних оптимізаційних математичних моделей, розрахунок значень температурного поля в ембріоні здійснений методами відокремлених змінних і невизначених коефіцієнтів. Застосувавши обчислені температури лазерного нагріву в шарах ембріона, реалізована математична модель мінімізації числа термічно-травмованих зародків.

Ключові слова: *варіативність, безпекове середовище, біооб'єкт, оптимізаційні математичні моделі, задачі математичного програмування.*